

* L. FLUID KINEMATICS AND DYNAMICS *

Fluid characteristics:

1) velocity (v)

2) acceleration (a)

3) density (ρ)

4) pressure (P)

Types of fluid flows:

1) steady flow and non-steady flow:

a) steady flow:

The fluid characteristics (or) properties does not changes with respect to time at any point of fluid is called steady state fluid flow.

$$\frac{\partial P}{\partial t} = \frac{dp}{dt} = \frac{dv}{dt} = 0$$

b) non-steady flow:

The fluid characteristics (or) properties (P, P, v, a) changes with respect to the time at any point of fluid is called unsteady flow.

flow

$$\frac{\partial P}{\partial t} \neq 0$$

$$\frac{\partial P}{\partial t} \neq 0$$

$$\frac{dv}{dt} \neq 0$$

(2) uniform flow and non-uniform flow:

a) uniform flow:

The velocity of fluid properties / fluid at any given time does not changes with respect to displacement is called uniform flow

$$\frac{dp}{ds} = 0$$

$$\frac{dv}{ds} = 0$$

$$\frac{dv}{ds} = 0$$

b) non-uniform steady fluid flow:

velocity of fluid properties at any time changes with respect to the

displacement is called non-uniform steady fluid flow

$$\frac{dp}{ds} \neq 0$$

$$\frac{dp}{ds} \neq 0$$

$$\frac{dv}{ds} \neq 0$$

(3) rotational and irrotational flows:

(a) Rotational flow:

The fluid particles moving in flow direction and rotates about

mass center (or) axis is called rotational flow.

(b) Irrotational flow:

The fluid particles does not moving in flow direction and does not

rotates about mass centre (or) axis is called irrotational flow.

(4) Laminar and turbulent flow:

(a) Laminar flow:

Fluid particle paths taken by individual particles and does not

crosses (or) intral another path lines such fluid flows are called

laminar flows.

fluid flow:
Fluid particles paths are in zig-zag and intercept with another path lines such field flows is called turbulent flow.

continuity equation:

which is the principle of conservation of mass is called continuity equation in fluid flow.

$$Q = V \cdot A$$

$$\frac{m^2 \cdot m}{s} = m^3/s$$

Potential function (or) velocity potential

which is the scalar function negative direction of space and time with respect in any direction (x, y, z) is called potential function of fluid particle denoted by ϕ .

fluid particle velocity in x -direction

$$U = -\frac{d\phi}{dx}$$

fluid particle velocity in y -direction

$$\frac{d\phi}{dy}$$

fluid particle velocity in z -direction

$$\frac{d\phi}{dz}$$

Stream function:

wolf liquid

which is derivative of space and time with respect to
any direction of velocity at a right angle to the flow direction
is called stream function.

→ It is denoted by ψ

fluid particle velocity in x-direction

$$v = \frac{\partial \psi}{\partial y}$$

fluid particle velocity in y-direction

$$v = -\frac{\partial \psi}{\partial x}$$

Flow lines:

which is a imaginary line along fluid particles flow

from up stream to down stream is called fluid lines.

Equipotential lines:

which is imaginary lines in fluid flow 1er direction, to
the flow lines is called equipotential lines

Flow nets:

which is the grid applied by series of flow lines and
equipotential lines obtained in mathematical analysis, graphi-
cal methods, and electrical analog method is called flownets.

continuity equation part 30: Now it will be derived

conservation of mass:

According to principle of conservation of mass, The increasing in fluid flow rate is equal to the change of mass flow rate in fluid.

$\rho = \text{density of the fluid}$

dx, dy, dz = are length of fluid elements

In x, y and z directions,

u, v, w are the velocity of fluid and x, y and z directions

mass flow rate at entrance of fluid element area in x -direction

$$X = \rho \cdot u \cdot dy \cdot dz \quad \text{--- (1)}$$

mass flow rate at exit of fluid element area in x -direction

$$X = \rho \cdot u \cdot dy \cdot dz + \frac{d}{dx} (\rho u \cdot dy \cdot dz) dx \quad \text{--- (2)}$$

The rate of change of fluid mass in x -direction eqn (1) - eqn (2)

$$= \rho \cdot u \cdot dy \cdot dz - \rho \cdot u \cdot dy \cdot dz - \frac{d}{dx} (\rho \cdot u \cdot dy \cdot dz) dx$$

$$X = - \frac{d}{dx} (\rho u \cdot dy \cdot dz) dx$$

$$= - \rho \frac{du}{dx} \cdot (dx \cdot dy \cdot dz) \quad \text{--- (3)}$$

change mass flow rate at exit fluid mass in y -direction.

$$Y = - \rho \cdot \frac{dv}{dy} \cdot (dx \cdot dy \cdot dz) \quad \text{--- (4)}$$

change mass flow in fluid element in z-direction

$$z = -\rho \frac{dw}{dz} dx dy dz \quad \text{--- (5)}$$

solve eqn (3), (4) and (5)

net mass flow rate in fluid element in 3 directions

$$x + y + z = -\rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dx dy dz \quad \text{--- (6)}$$

increase of mass in x, y, z directions.

$$\frac{\partial \rho}{\partial t} \cdot dx dy dz \quad \text{--- (7)}$$

According to law of conservation of mass

$$\text{eqn (6)} = \text{eqn (7)}$$

$$-\rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dx dy dz = \frac{\partial \rho}{\partial t} \cdot dx dy dz$$

$$\frac{\partial \rho}{\partial t} = -\rho$$

$$dx dy dz - \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) - \frac{\partial \rho}{\partial t} dx dy dz = 0$$

$$\rho \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] + \frac{\partial \rho}{\partial t} = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{1}{\rho} \frac{\partial \rho}{\partial t} = 0}$$

i) for steady state fluid flow $\frac{\partial \rho}{\partial t} = 0$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

continuity equation in 3 dimension.

$$\frac{du}{dx} + \frac{du}{dy} = 0$$

continuity equation in 2-D

$$\boxed{\frac{du}{dx} = 0}$$

continuity equation in 1-D

problem:

The fluid flow is $\mathbf{v} = x^2 y \mathbf{i} + y^2 z \mathbf{j} - (2xyz + yz^2) \mathbf{k}$ prove that

it is a possibility of fluid flow and calculate velocity at (2, 1, 3)

sol:

Given equation $\mathbf{v} = x^2 y \mathbf{i} + y^2 z \mathbf{j} - (2xyz + yz^2) \mathbf{k}$

this is in the form of

$$\mathbf{v} = \bar{u} \mathbf{i} + \bar{v} \mathbf{j} + \bar{w} \mathbf{k}$$

where $u = x^2 y \quad v = y^2 z \quad w = -(2xyz + yz^2)$

$$\frac{\partial u}{\partial x} = 2xy \quad \frac{\partial v}{\partial y} = 2yz \quad \frac{\partial w}{\partial z} = -(2xy + 2yz)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2xy + 2yz - 2xy - 2yz$$

$$= 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

Satisfies the continuity equation of flow is possible

$$\mathbf{v} = x^2 y \mathbf{i} + y^2 z \mathbf{j} - (2xyz + yz^2) \mathbf{k} \quad (1, 2, 3)$$

$$= (2)^2 (1) \mathbf{i} + (1)^2 (3) \mathbf{j} - [2(2)(1)(3)] + (1)(3)^2 \mathbf{k}$$

$$\mathbf{v} = 4 \mathbf{i} + 3 \mathbf{j} - 21 \mathbf{k}$$

$$v = \sqrt{u^2 + v^2 + w^2}$$

$$= \sqrt{(4)^2 + (3)^2 + (21)^2}$$

unit - 2, Pg - 7/17

$$V = 21.58 \text{ m/s}$$

$$\rho = \frac{\rho b}{\rho b + \rho D}$$

Bernoulli's equation (Euler's equation) :

$$\rho + \frac{\rho b}{2} = \text{constant}$$

Bernoulli's theorem

In ideal compressible fluid in steady and continuous flow, the summation of pressure energy, kinetic energy and potential energy is constant in fluid is called Bernoulli theorem (or) Bernoulli equation.

ρ = density of fluid

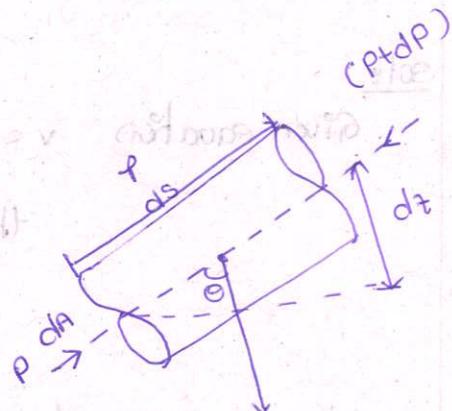
P = pressure in fluid

ds = element length

dA = element area

V = velocity of fluid

dz = bottom length



pressure at inlet of fluid element P

pressure at exit of fluid element $P+dp$

pressure force at inlet = ρdA \rightarrow 0

pressure force at exit in fluid element = $-(P+dp)dA$

$$\rho = \frac{\rho b}{\rho b + \rho D} + \frac{\rho b}{\rho b + \rho D} + \frac{\rho b}{\rho b + \rho D} \quad \rightarrow ②$$

net pressure force in fluid element

$$P \cdot dA - PdA - dp \cdot dA = -dp \cdot dA$$

$$P = \frac{dp \cdot dA}{dA} \quad \rightarrow ③$$

weight of fluid element in flow direction

$$= \rho g dA ds (\sin \theta) \quad \rightarrow ④$$

$$= \rho g dA ds \cos \theta$$

Acceleration of fluid flow

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$= \frac{dv}{ds} \cdot v \left(\frac{ds}{dt} \right) = v \quad \text{--- (4)}$$

From eqn (1) to (4) we get

$$\frac{dp}{ds} = -\rho g da \Rightarrow \frac{dp}{ds} = -\rho g v \frac{dv}{ds} \quad \text{--- (5)}$$

From equation (3) (4) (5)

$$\Sigma F = ma$$

$$-\frac{dp}{ds} - \rho g \frac{dv}{ds} = m \cdot v \cdot \frac{dv}{ds}$$

$$-\frac{dp}{ds} - \rho g v \frac{dv}{ds} - \rho v \cdot dv = 0 \quad \text{--- (6)}$$

equation (6) divided by ρg

$$\boxed{\frac{\frac{dp}{ds}}{\rho g} + v \cdot \frac{dv}{g} + dz = 0}$$

equation (5) is called as Eular's equation

applying integration to eular's equation

$$= \int \frac{dp}{\rho g} + v \int \frac{dv}{g} + \int dz = \int_0$$

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = c}$$

Bernoulli equation at any two points

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2}$$

continuity equation in cartesian coordinate for one-dimension flow:

The conservation of mass energy of mass states that the mass is neither be created nor be destroyed. When this law is applied to a fluid system with boundary conditions (i.e. under control volume) then it states that, the rate of accumulation of mass within control volume is equal to the rate of increase of flow in control volume

$$\text{i.e. } \frac{d}{dt} (m)_{cv} = M_{in} - M_{out}$$

where

M_{in} = Rate of mass inflow

M_{out} = Rate of mass outflow

For steady flow rate of change of fluid mass within the control volume is zero

$$\therefore M_{in} = M_{out}$$

one-dimensional continuity equation:

consider an one dimensional infinitesimal stream flow in a differential control volume between section ① and ②

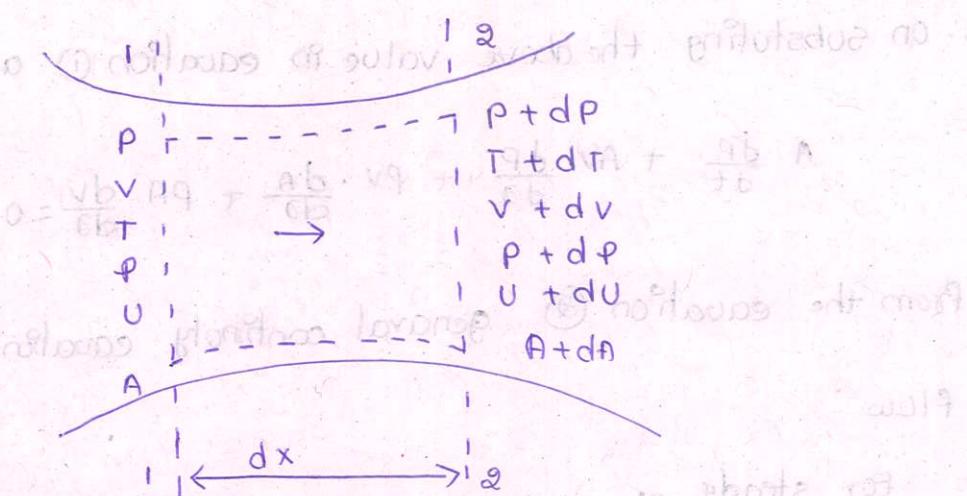
at section ① the fluid with pressure (P) temperature (T) velocity (v)

density (ρ) and internal energy (u) be flowing through a cross-section.

A corresponding properties at section 2, has changed to pressure

($P+da$) temperature ($T+dt$) velocity ($v+dv$) density ($P+d\rho$)

and internal energy ($u+du$) through a cross-section ($A+da$)



The equation of mass conservation of control volume is given as

$$\frac{d}{dt} (m)_{cv} = m_{in} - m_{out}$$

Since mass of the fluid within control volume is the product of average density and volume i.e $m_{cv} = \rho A \cdot dx$

Rate of mass inflow for the control volume at section 1 can be

written as

$$m_{in} = \rho A V$$

Rate of mass outflow for the control volume at section 2 can be written as

$$m_{out} = \rho A V$$

Rate of flow mass outflow for the control volume at section 2, can be written as

$$m_{out} = \rho A V$$

Rate of mass outflow for the control volume at section 3 can be written as

$$m_{out} = (P + dP) \cdot (A + dA) \cdot (V + dV)$$

$$= \rho A V + \rho A dV + \rho V dA + A dP + \text{High Order terms}$$

$$\therefore m_{in} - m_{out} = \rho A V - [\rho A V + \rho A dV + \rho V dA + A dP]$$

∴ On substituting the above value in equation ① and dividing with dx

$$A \cdot \frac{dp}{dt} + AV \cdot \frac{dp}{da} + PV \cdot \frac{dA}{da} + PA \cdot \frac{dv}{da} = 0 \quad \text{--- ②}$$

From the equation ② general continuity equation for one dimensional flow

For steady flow, there is no variation of fluid flow properties with time i.e. $\frac{dp}{dt} = 0$

$$\therefore \text{The equation ② becomes } A V \cdot \frac{dp}{da} + P V \cdot \frac{dA}{da} + P A \cdot \frac{dv}{da} = 0$$

$$\therefore \text{The equation ③ becomes } \frac{dp}{da} + V \frac{dA}{da} + A V \frac{dv}{da} = 0 \quad \text{--- ③}$$

$$AV \cdot \frac{dp}{da} + PV \cdot \frac{dA}{da} + PA \cdot \frac{dv}{da} = 0$$

$$\therefore \frac{d}{da} (PAV) = 0$$

∴ The continuity equation for steady flow is $PAV = \text{constant}$

(i) Determine whether the following velocity components satisfy the continuity equation.

$$(i) u = 2x^2 + zy$$

$$v = -2xy + 3y^2 + 3zy$$

$$w = -\frac{3}{2}z^2 - 2xy - 6yz$$

$$(ii) u = -\frac{c}{y}$$

$$v = c \log xy$$

$$(i) u = 2x^2 + zy \quad v = -2xy + 3y^2 + 3zy \quad w = -\frac{3}{2}z^2 - 2xy - 6yz$$

$$\frac{\partial u}{\partial x} = 4x \quad \frac{\partial v}{\partial y} = -2x + 9y^2 + 3z \quad \frac{\partial w}{\partial z} = -3z - 6y$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = (4x) + (-2x + 9y^2 + 3z) + (-3z - 6y)$$

$$[4x + 9y^2 - 6y] \neq 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \neq 0$$

But according to continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Hence, the given velocity components does not satisfies the continuity equation.

(ii) $u = -\frac{cx}{y}$ $v = c \log z y$

$$u = -\frac{cx}{y} \quad v = c \log z y$$

$$\frac{\partial u}{\partial x} = -\frac{c}{y} \quad \frac{\partial v}{\partial y} = -\frac{cx}{xy} = -\frac{c}{y}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{c}{y} + \frac{c}{y} = 0$$

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0$$

The equation of continuity is satisfied.

Various forces influence the motion of fluid:

The various forces that may influence the motion of fluid are as follows.

1) elastic force (F_e)

2) gravitational force (F_g)

3) inertia force (F_i)

4) pressure force (F_p)

5) surface tension force (F_s)

6) viscous force (F_v)

1) Elastic force (F_e)

The product of elastic stress and area of the following fluid is known as elastic force.

2) gravity force (F_g)

The product of mass and acceleration due to gravity is known as gravity force. This force exist in open surface flow.

3) inertia force (F_i)

The force at which acts in the opposite direction of acceleration.

The product of the mass and acceleration of the motion of fluid is termed as inertia force.

4) pressure force (F_p)

The product of pressure intensity and cross sectional area of the following fluid is known as pressure force. This force exists in pipe flows.

5) surface tension force (F_s)

The product of surface tension and length of surface of the following fluid is known as surface tension force.

6) viscous forces (F_v)

The product of surface area of the flow and the shear stress due to viscosity is known as viscous forces.

Applications of Bernoulli's Equation:

It is applied in all problems of incompressible fluid

Flow where energy considerations are involved, but here we consider its application to the flowing measuring devices.

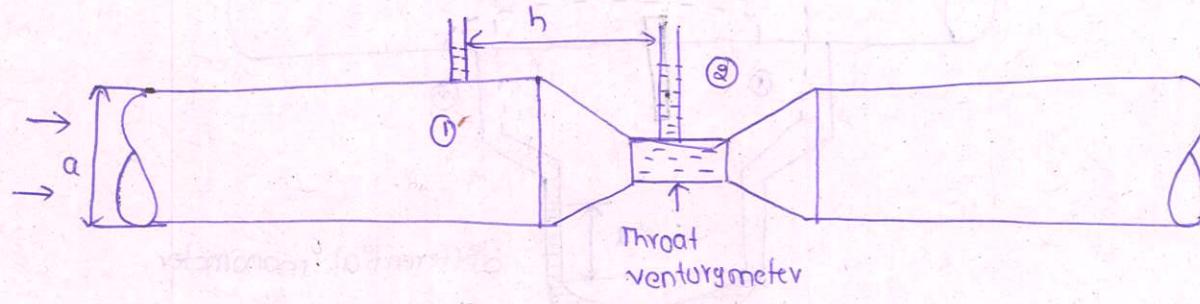
1) Venturiometer

2) Orifice meter

3) Pitot tube

1) Venturiometer

It is used to measure the rate of a flow of a fluid flowing through a pipe



Let

d_1 and d_2 - diameter of inlet or at section (1) and (2)

P_1 and P_2 - pressure at section (1) and (2)

v_1 and v_2 - velocity of fluid at section (1) and (2)

$$A = \text{Area of section} = \frac{\pi}{4} d^2$$

According to Bernoulli's equation the total energy at any point of the fluid at any point of the fluid is constant, i.e. sum of pressure energy, kinetic energy and potential energy

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

(2) orifice meter:

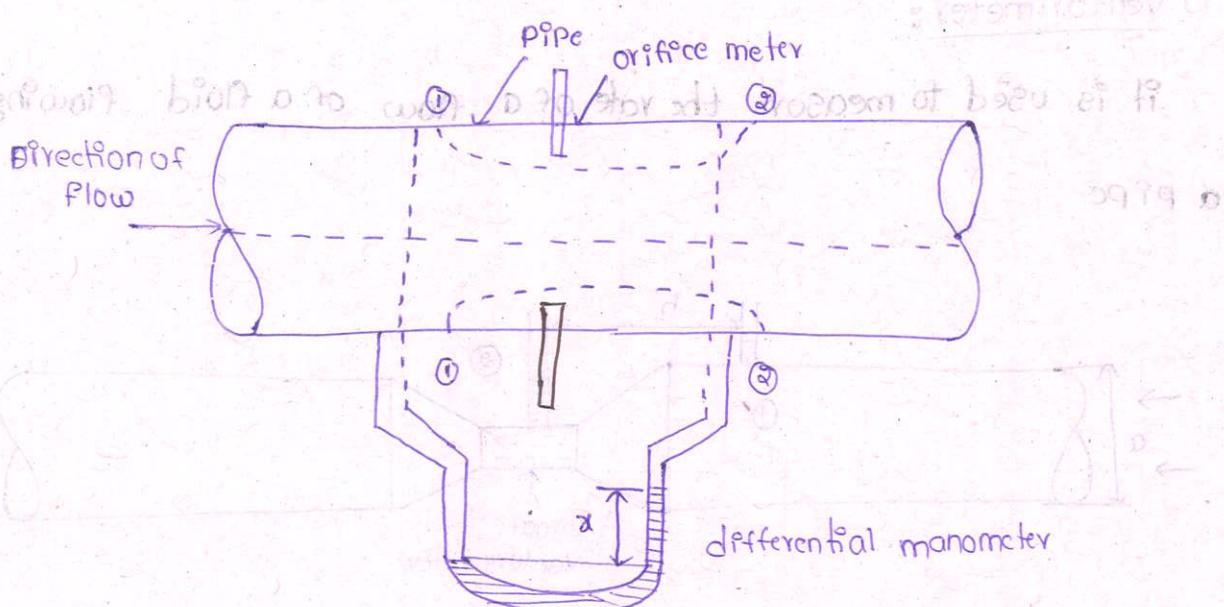
An orifice meter is a discharge measurement device within the pipes. The cross section view of orifice meter is shown in the figure (2)

Let

p_1, p_2 - pressure at section ① and section ②

v_1, v_2 - velocity at section ① and ②

A_1, A_2 - Area of pipe section at ① and ②



on applying Bernoulli's equation at section ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

3) pitot tubes:

It is used to measure the velocity of flow at any point in

a pipe or a channel. Here also Bernoulli's equation is applied

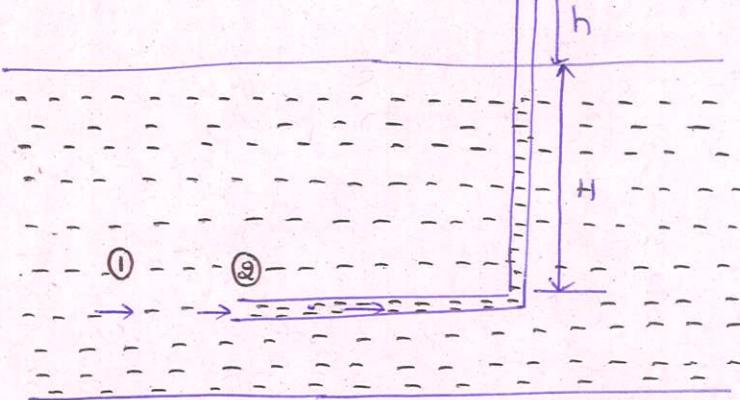
Let

P_1, P_2 - intensity of pressure at point (1) and (2)

V_1, V_2 - velocity of flow at ① and ②

H_B - depth of the tube in the liquid.

on



applying Bernoulli's equation at points (1) and (2)

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2}$$

